Public Safety under Imperfect Taxation*

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Abstract

In this paper, we examine theoretically the effect of tax system imperfections on the optimal public investment in mortality risk reduction (or public safety). We compare three exogenous tax systems, namely first-best, uniform tax and income tax. Moreover, we consider several sources of imperfections, namely individuals' heterogeneity in wealth and in risk exposure, and labor supply distortion. We show that the direction of the effect of imperfect taxation critically depends on the source of imperfection as well as on the utility and on survival probability functions. We conclude that imperfect taxation cannot generically justify more or less public safety. There is thus no fundamental reason to always adjust downwards the value of statistical life (VSL) because of imperfect taxation, nor to assume a marginal cost of public funds systematically greater than one for the benefit-cost analysis of public safety projects.

Keywords: Safety provision, imperfect taxation, value of statistical life, distortionary taxation, wealth inequality, risk aversion.

JEL Classification: D61, H21, H41, I18, Q51

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1 Introduction

Mortality reduction represents a significant part of the benefit of many public projects. It has been estimated to account for more than 90% of the monetized benefit of the Clean Air Act (U.S. Environmental Protection Agency, 2011). In practice, the standard approach to compute this benefit is to multiply the number of expected lives saved by the average value of statistical life (VSL) in the affected population.[1] This standard practice in benefit-cost analysis (BCA) is justified theoretically under the assumption that the taxation system is "perfect", namely a system based on individualized lump-sum taxes. In this paper, we relax this assumption and examine how the imperfections of the taxation system affect the optimal level of public safety, and in turn whether adjustments in the standard VSL approach are warranted.

Accounting for imperfect taxation in the evaluation of public safety projects is important for several reasons. First, it is well documented that the taxation system is imperfect in both developed and developing countries and that the degree of imperfection varies widely across the world (Tanzi and Zee, 2001).[2] Second, from a policy perspective, various guidelines encourage policy evaluations to also include in BCA "distributive impacts", "equity", or "environmental justice" (European Commission, 2009; U.S. Environmental Protection Agency, 2016a). But it is also well known that concrete methodologies for evaluating such additional impacts remain undeveloped (Adler, 2008). Moreover, safety issues usually raise strong equity concerns that call for a careful and systematic analysis of distributive impacts. Third, the literature in public economics has long debated in general settings the issue of the optimal provision of public goods under distortionary taxes and individual heterogeneities (Atkinson and Stiglitz, 1980). It thus seems useful to examine a specific but important domain of application such as safety provision. A starting point to do so is to develop a comparative statics analysis of the effect of taxation system imperfections on the optimal level of public safety.

In our analysis, we proceed as follows. We compare the optimal level of public safety selected by a utilitarian social planner under three exogenous benchmark types of the taxation system: individual lump-sum tax (first-best), uniform lump-sum tax (uniform tax) and uniform flat tax (income tax).[3] We consider in turn two types of individual heterogeneity, namely wealth and mortality risk heterogeneity, and we also consider distortionary taxation. Our primary results are the following. Under wealth heterogeneity, compared with the first-best level of public safety, we show that the optimal level of public safety provision is usually lower under uniform taxation, but that it can be greater under income taxation. Under mortality risk heterogeneity, we show that the comparison depends on whether the heterogeneity concerns the baseline risk or the reduction in risk. Specifically, under heterogeneous baseline risk, public safety in the first-best can be lower or higher than under either income or uniform taxation depending on the utility function. Under heterogeneity in risk reduction, public safety is in general greater in the first-best than under uniform or income taxation. Finally, we show that there can be more or less

[1] Environmental Protection Agency (EPA) guidelines recommend using a VSL of $9.7 million in 2013 U.S dollars (U.S. Environmental Protection Agency, 2010). In 2016, the U.S. Department of Transportation (DOT) uses a VSL of $9.6 million for their analyses (U.S. Department of Transportation, 2016).

[2] For example, according to OECD (2017), Hungry still implements a flat income tax system, whereas other OECD countries implement a progressive tax system. However, the degree of progressivity varies widely across countries.

[3] In the optimal taxation literature, endogenous taxation is typically studied to account for the issue of imperfect information (Mirrlees, 1971). In our setting, we assume for simplicity that the tax system is exogenously given.

public safety under first-best compared to distortionary taxation depending on whether the labor effort is "tangible" (i.e., commensurable with wealth) and on the shape of the utility function.

From this theoretical analysis, we conclude that the imperfection of the taxation system cannot generically justify more or less public safety provision. The basic intuition is simple. Take the wealth heterogeneity case for example. Under perfect taxation, the rich is taxed more than the poor. Imperfect taxation shifts some of the tax burden from the rich to the poor. Thus, the rich is relatively richer and would prefer more public safety, whereas the poor is relatively poorer and would prefer less public safety. Depending on the shape of the utility function, the change in preference and thus the demand for safety of the rich may, or may not, over shadow that of the poor, so that more or less safety should be provided.

Our paper builds on two strands of literature: the VSL and the optimal provision of public good literature. First, the VSL represents the individual's marginal willingness to pay for a small reduction in mortality risk (Drèze, 1962; Jones-Lee, 1974). The VSL literature has examined both theoretically and empirically how VSL varies with the characteristics of individuals or of the decision-making environment (Andersson and Treich, 2011; Viscusi and Aldy, 2003). However, the vast majority of this literature has ignored the issue of imperfect taxation, with two notable exceptions. Pratt and Zeckhauser (1996) study the optimal allocation of safety among heterogeneous individuals under uniform taxation.[4] Armantier and Treich (2004) examine the bias induced by the standard VSL approach under uniform taxation when individuals are heterogeneous in wealth and mortality risk. In this paper, we compare under the social welfare framework the optimal level of public safety under different taxation systems combined with individual heterogeneity in wealth or risk to identify the bias introduced by imperfect taxation.

Second, in the public good provision literature, a standard reference is the Pigou conjecture. Pigou (1947) states that, under distortionary taxation, the marginal benefit of the public good should be greater than the marginal production cost, implying a lower provision of the public good.[5] This conjecture led to the development of the marginal cost of public funds (MCPF) concept, which was first incorporated into the Samuelson's rule for the optimal public good provision by Stiglitz and Dasgupta (1971), Diamond and Mirrlees (1971) and Atkinson and Stern (1974). If Pigou's conjecture holds, the value of MCPF should be greater than 1. However, the literature has shown that this conjecture holds only under specific settings and that the value of the MCPF depends on the relationship between the public good, labor supply, and the taxed activities (Stiglitz and Dasgupta, 1971; Atkinson and Stern, 1974; Ballard and Fullerton, 1992). Gaube (2000) shows for instance that with heterogeneous households, equity consideration may increase public expenditure in the second best. In practice, BCA typically recommends using an MCPF larger than one to account for imperfect taxation, which seems questionable given the lack of consensus in the literature. Moreover, although the MCPF has been extensively studied, we are not aware of any specific application to public safety. Besides, contrary to the standard assumption of additive separability

[4] Pratt and Zeckhauser (1996) focus on the collectively purchased risk reduction that can be targeted at different individuals. Under uniform taxation, they study the optimal individual safety level, whereas we study the optimal public safety (i.e. individuals consume the same amount of safety).

[5] Pigou (1947, p.33-34) noted: "Where there is indirect damage, it ought to be added to the direct loss of satisfaction involved in the withdrawal of the marginal unit of resources by taxation, before this is balanced against the satisfaction yielded by the marginal expenditure. It follows that, in general, expenditure ought not to be carried so far as to make the real yield of the last unit of resources expended by the government equal to the real yield of the last unit left in the hands of the representative citizen."

between the public good and private consumption (Atkinson and Stiglitz, 1980; Wilson, 1991; Gaube, 2000), public safety typically has a non-additive-separability characteristic between the two, as the public good enters the expected utility function multiplicatively through the survival probability (see 2.1 below). Moreover, most of the literature in public economics examines the level and properties of optimal public goods provision under a specific taxation system, but does not compare various systems. Thus, we contribute to this literature by examining the optimal provision under the public safety framework.

2 The Model

In this section, we set up the benchmark model of optimal public safety provision. We consider a single period economy with H individuals that differ only in wealth w_i and mortality risk $1 - p_i$ $(i = 1, ..., H)$. We assume that the utility function is uniform across individuals and the bequest motive is normalized to zero. Following the VSL literature, the individual i's expected utility is given by

$$EU_i = p_i(G)u(c_i, l_i) \tag{2.1}$$

Here $p_i(G)$ denotes the probability of survival given the level of public expenditure on safety G. $u(\cdot)$ is the individual's survival utility as a function of her consumption level c_i and labor supply l_i. Under an exogenous wage rate ω_i, individual i has wealth $w_i \equiv \omega_i l_i$ and when the individual faces a tax rate t_i, the consumption level is $c_i = w_i - t_i$.

In this model, the utility function is assumed to be strictly positive $(u > 0)$, since the bequest motive is normalized to zero and survival is assumed to be strictly preferred to death.[6] The utility function is increasing and concave in the consumption level $(u_c > 0,\ u_{cc} < 0)$, and decreasing and concave in labor supply $(u_l < 0,\ u_{ll} < 0)$.

We assume that the survival function is positive, increasing, and weakly concave $(p_i(\cdot) > 0,\ p_i'(\cdot) > 0,\ p_i''(\cdot) \leq 0)$, and $p_i(G) < 1$ for all is. For the simulations, we will use the functional form $p_i(G) = a_i + b_i \frac{G}{1+G}$, where $0 \leq a_i < 1 - b_i$ and $0 \leq b_i \leq 1$.[7]

2.1 First-best Benchmark

In the first-best, the utilitarian social planner chooses the optimal level of public safety G and the lump-sum tax rate t_i (a subsidy is a negative tax) for each individual i by solving the following welfare

[6] The possibility of a bequest motive $v(c_i, l_i)$ can also be considered, then $EU_i = p_i(G)u(c_i, l_i) + (1 - p_i(G)v(c_i, l_i)$. As is common in the literature, we could consider the case that $v(c_i, l_i) = ku(c_i, l_i)$ for some k ($k \in [0, 1)$ for $u > 0$ and $k > 1$ for $u < 0$) (Kaplow, 2005; Viscusi and Evans, 1990). This means that the utility in the death state is proportionally lower than the survival utility. Therefore, for each individual, we can write $\pi_i(G) = k + (1 - k)p_i(G)$, $\pi_i(\cdot) > 0$, $\pi_i'(\cdot) > 0$, $\pi_i''(\cdot) \leq 0$, and $EU_i = \pi_i(G)u(c_i, l_i)$. It is straightforward that all results of the paper carry out under this particular case.

[7] When the specific functions are used, we would verify that the optimal level of public safety $G^* > 0$.

maximization problem:

$$\max_{G,\{t_i\}_{i\in\{1,\dots H\}}} \quad \sum_{i=1}^{H} p_i(G)u(c_i, l_i)$$
$$\text{where} \quad c_i = \omega_i l_i - t_i \quad \forall i \tag{2.2}$$
$$\text{s.t.} \qquad G \leq \sum_{i=1}^{H} t_i$$

Setting the Lagrangian:

$$\mathcal{L} = \sum_{i=1}^{H} p_i(G)u(\omega_i l_i - t_i, l_i) + \mu(\sum_{i=1}^{H} t_i - G) \tag{2.3}$$

the first order conditions (focs) with respect to t_i and G give

$$\frac{\partial \mathcal{L}}{\partial t_i} = p_i(G^*)u_c(\omega_i l_i - t_i^*, l_i) - \mu = 0, \quad \forall i \tag{2.4}$$

$$\frac{\partial \mathcal{L}}{\partial G} = \sum_{i=1}^{H} p_i'(G^*)u(\omega_i l_i - t_i^*, l_i) - \mu = 0, \tag{2.5}$$

where μ denotes the shadow price of one additional unit of public safety.[8] The focs indicate that the social planner equalizes the after tax expected marginal utility of wealth across individuals. As the tax levied on each individual is lump-sum, the individual's labor supply is not distorted and the optimal decision $c_i^*(t_i^*)$, $l_i^*(t_i^*)$ satisfies: $-\frac{u_l^*}{u_c^*} = \omega_i$, where $u_c^* \equiv \frac{\partial u(c_i^*(t_i^*), l_i^*(t_i^*))}{\partial c_i}$ and $u_l^* \equiv \frac{\partial u(c_i^*(t_i^*), l_i^*(t_i^*))}{\partial l_i}$.

Replacing μ in equation 2.5 and rearranging, we get

$$\sum_{i=1}^{H} p_i'(G^*)VSL_i = 1 \tag{2.6}$$

$$\text{where} \quad VSL_i \equiv \frac{u^*}{p_i(G^*)u_c^*}$$

with $u^* \equiv u(c_i^*(t_i^*), l_i^*(t_i^*))$ and $G^* = \sum_{i=1}^{H} t_i^*$. VSL_i is the value of statistical life (VSL) of individual i, which describes the marginal rate of substitution (MRS) between wealth and survival probability. VSL exhibits two standard effects, namely the dead-anyway effect and the wealth effect. The dead-anyway effect (Pratt and Zeckhauser, 1996) states that VSL decreases in the survival probability p_i, i.e. the individual facing higher risks has the incentive to increase his spending on risk reduction. The wealth effect states that VSL increases in the individual's disposable wealth $w_i - t_i$.

Equation 2.6 characterizes the efficiency condition to achieve the optimal level of public safety provision. It corresponds to the Samuelson's condition (Samuelson, 1954) of equalizing social marginal benefit to the social marginal cost of providing for the public good.

[8] We will assume throughout that the second order conditions hold globally. See appendix A.1 for more details.

2.2 Second-best Linear Taxation

We denote the first-best tax system as "perfect taxation", as there is redistribution without distortion. In the this section, we characterize the optimal safety provision under linear taxation. $T(w_i) = t + \tau w_i$ is the linear tax schedule with t denoting the uniform lump-sum tax level and τ denoting the flat tax rate.

For each individual facing the linear tax schedule, she maximizes her expected utility subject to the new budget constraint $c_i = (1-\tau)\omega_i l_i - t$. The optimal labor supply and consumption is thus characterized by $-\frac{u_l^*}{u_c^*} = (1-\tau)\omega_i$.

The social planner thus maximizes social welfare choosing the optimal public safety level, lump-sum tax level and flat tax rate:

$$\begin{aligned} \max_{G,t,\tau} \quad & \sum_{i=1}^{H} p_i(G)u((1-\tau)\omega_i l_i^*(\tau) - t, l_i^*(\tau)) \\ \text{s.t.} \quad & \sum_{i=1}^{H}(t + \tau\omega_i l_i^*(\tau)) = G \end{aligned}$$

Setting the Lagrangian:

$$\mathcal{L} = \sum_{i=1}^{H} p_i(G)u((1-\tau)\omega_i l_i^*(\tau) - t, l_i^*(\tau)) + \mu(\sum_{i=1}^{H}(t + \tau\omega_i l_i^*(\tau)) - G) \tag{2.7}$$

the focs with respect to G, t and τ give

$$\frac{\partial \mathcal{L}}{\partial G} = \sum_{i=1}^{H} p_i'(G^*)u((1-\tau^*)\omega_i l_i^*(\tau) - t^*, l_i^*(\tau)) - \mu = 0, \tag{2.8}$$

$$\frac{\partial \mathcal{L}}{\partial t} = -\sum_{i=1}^{H} p_i(G^*)u_c((1-\tau^*)\omega_i l_i^*(\tau) - t^*, l_i^*(\tau)) + \mu H = 0 \tag{2.9}$$

$$\frac{\partial \mathcal{L}}{\partial \tau} = -\sum_{i=1}^{H} p_i(G^*)u_c((1-\tau^*)\omega_i l_i^*(\tau) - t^*, l_i^*(\tau))\omega_i l_i^*(\tau) + \mu\sum_{i=1}^{H}\omega_i l_i^*(\tau)(1+\varepsilon_i) = 0 \tag{2.10}$$

where $\varepsilon_i = \frac{\partial l_i}{\partial \tau}/\frac{l_i}{\tau}$ denotes the individual's labor supply elasticity of income tax.

Using obvious notations, equation 2.8 and 2.9 imply that

$$\sum_{i=1}^{H} p_i'(G^*)u^* = \frac{1}{H}\sum_{i=1}^{H} p_i(G^*)u_c^* \tag{2.11}$$

which shows with uniform lump-sum tax, optimality is achieved when the social planner equalizes the social marginal benefit of public safety (LHS) with the average marginal cost (RHS).

Similarly, equation 2.8 and 2.10 imply that

$$\sum_{i=1}^{H} p_i'(G^*)u^* = \frac{\sum_{i=1}^{H} p_i(G^*)u_c^*\omega_i l_i^*(\tau)}{\sum_{i=1}^{H}\omega_i l_i^*(\tau)(1+\varepsilon_i)} \tag{2.12}$$

which shows with uniform flat tax, the optimality condition equalizes the social marginal benefit (LHS) with the income and elasticity weighted marginal cost.

Therefore, the second-best optimal public safety would depend on the taxation instrument, the individual survival functions $p_i(\cdot)$, the utility function $u(\cdot)$, and individual labor supply elasticity ε_i. However, it is hard to compare directly the public safety level in the first-best and in the second-best under this general setting.

To disentangle the effect of different taxation schemes and sources of imperfection, we carry out the analyses with one variation at a time. In particular, it will turn out to be fruitful for the comparative static analysis to systematically compare three benchmark tax systems: individual lump-sum taxes (first-best), uniform lump-sum taxes (uniform tax), and uniform flat taxes (income tax). In section 3 and 4, we study the case of imperfect redistribution between heterogeneous individuals with exogenous labor supply and we denote without loss of generality that $u(c_i) \equiv u(c_i, l_i)$. To illustrate the analysis, two common utility forms will be used, namely constant relative risk aversion (CRRA) utility and constant absolute risk aversion (CARA) utility: with CRRA utility, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \gamma \in (0,1)$; with CARA utility, $u(c) = \frac{1-e^{-\alpha c}}{\alpha}, \alpha > 0$.[9] Two analytically important coefficients are relative risk aversion $R(c) = -c\frac{u''(c)}{u'(c)}$ and the fear of ruin $FR(c) = \frac{u(c)}{u'(c)}$ (Foncel and Treich, 2005). The only class of utility function that has linear fear of ruin is CRRA, which also has $R(c) = \gamma$. In section 5, we study the case of labor effort distortion with income taxes on identical individuals with utilities $u(c, l)$.

3 Wealth Heterogeneity

In this section, we examine the case of individual heterogeneity in wealth. In the first-best, tax t_i is levied on individual i. Deviating from the first-best, we consider two cases: uniform tax t_U and income tax τw_i. For simplicity, we assume $H = 2$. Thus, the social planner solves the following maximization problems under the three tax systems:

First-best:

$$\begin{aligned} \max_{G_F, t_1, t_2} \quad & p(G_F)[u(w_1 - t_1) + u(w_2 - t_2)] \\ \text{s.t.} \quad & G_F = t_1 + t_2 \end{aligned} \tag{3.1}$$

Uniform Tax:

$$\begin{aligned} \max_{G_U, t_U} \quad & p(G_U)[u(w_1 - t_U) + u(w_2 - t_U)] \\ \text{s.t.} \quad & G_U = 2t_U \end{aligned} \tag{3.2}$$

[9]Note that because we assume $u > 0$, we impose $\gamma < 1$ and $\alpha > 0$. This assumption restricts the class of CRRA and CARA utility functions that we consider in this paper.

Income Tax:

$$\begin{aligned} \max_{G_I,\tau} \quad & p(G_I)[u(w_1(1-\tau)) + u(w_2(1-\tau))] \\ \text{s.t.} \quad & G_I = \tau(w_1 + w_2) \end{aligned} \tag{3.3}$$

Rearranging the focs, we can easily get the following equations:

First-best:

$$\frac{p(G_F^*)}{p'(G_F^*)} = \frac{u(w_1 - t_1^*)}{u'(w_1 - t_1^*)} + \frac{u(w_2 - t_2^*)}{u'(w_2 - t_2^*)} \tag{3.4}$$

Uniform Tax:

$$\frac{p(G_U^*)}{p'(G_U^*)} = 2\frac{u(w_1 - t_U^*) + u(w_2 - t_U^*)}{u'(w_1 - t_U^*) + u'(w_2 - t_U^*)} \tag{3.5}$$

Income Tax:

$$\frac{p(G_I^*)}{p'(G_I^*)} = (w_1 + w_2)\frac{u(w_1(1-\tau^*)) + u(w_2(1-\tau^*))}{w_1 u'(w_1(1-\tau^*)) + w_2 u'(w_2(1-\tau^*))} \tag{3.6}$$

Note that equation 3.4 corresponds to 2.6 and that equations 3.5 and 3.6 correspond to 2.11 and 2.12 respectively. The focs of equation 3.1 imply $w_1 - t_1^* = w_2 - t_2^*$. Assuming $w_1 > w_2$, we can infer that $t_1^* > t_2^*$. Thus under wealth heterogeneity, the first-best requires a higher tax on the wealthier individual. This is in line with the wealth effect on VSL. In the remainder of this section, we separately compare first-best G_F^* with uniform tax G_U^* and with income tax G_I^*.

3.1 First Best and Uniform Tax Comparison

Proposition 1. *Under wealth heterogeneity, with $u'''(x) \geq 0$, the optimal level of public safety in the first-best is higher than that with uniform taxation ($G_F^* > G_U^*$).*

Proof. The focs of the first-best and uniform tax maximization problems equalize the marginal benefit of public safety to its marginal cost of provision. Thus equations 3.4 and 3.5 can be rewritten as

$$p'(G_F^*)[u(w_1 - t_1^*) + u(w_2 - t_2^*)] = p(G_F^*)[\frac{1}{2}u'(w_1 - t_1^*) + \frac{1}{2}u'(w_2 - t_2^*)] \tag{3.7}$$

$$p'(G_U^*)[u(w_1 - \frac{G_U^*}{2}) + u(w_2 - \frac{G_U^*}{2})] = p(G_U^*)[\frac{1}{2}u'(w_1 - \frac{G_U^*}{2}) + \frac{1}{2}u'(w_2 - \frac{G_U^*}{2})] \tag{3.8}$$

The left-hand side (LHS) for both equations 3.7 and 3.8 can be regarded as the marginal benefit and the right-hand side (RHS) as the marginal cost. Observe that for any given level of $G = t_1 + t_2$ such that $w_1 - t_1 = w_2 - t_2$, risk aversion implies that

$$u(w_1 - t_1) + u(w_2 - t_2) > u(w_1 - \frac{G}{2}) + u(w_2 - \frac{G}{2}),$$

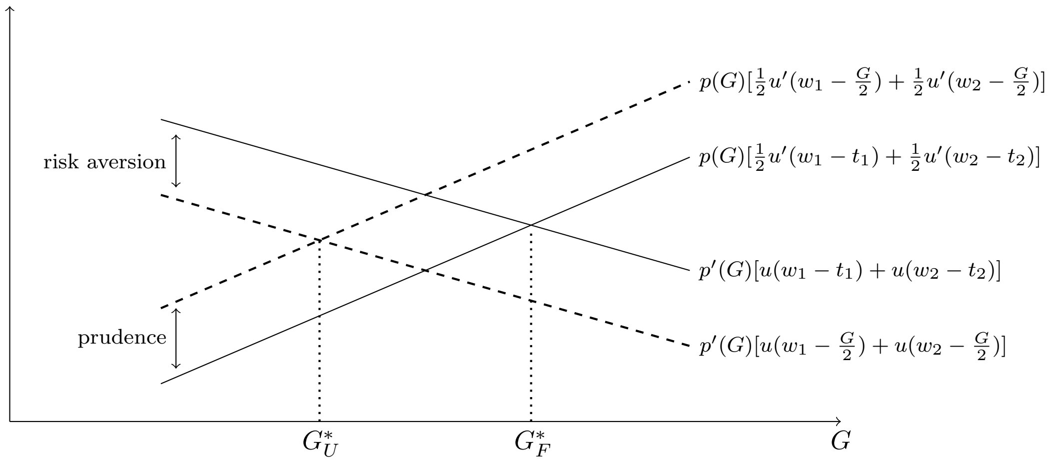
and under prudence $u''' \geq 0$, we have

$$u'(w_1 - t_1) + u'(w_2 - t_2) \leq u'(w_1 - \frac{G}{2}) + u'(w_2 - \frac{G}{2}).$$

Therefore, for the same level of G, the LHS of equation 3.7 is greater than that of 3.8 and the RHS of

3.7 is lower than that of 3.8. As Figure 1 illustrates, under risk aversion and prudence, we must have $G_F^* > G_U^*$ at the optimum.

Fig. 1 Illustration of the comparison between first-best and uniform tax



□

Proposition 1 shows that, under the common assumption of prudence (Kimball, 1990), where the marginal utility of consumption is decreasing at a diminishing rate, the optimal level of public safety in the first-best is higher than that under uniform tax.

The intuition for this result is the following. In the first-best, the social planner equalizes the individual marginal utilities. When perfect taxation is not possible, the marginal utility cannot be equalized which affects both the marginal benefit and marginal cost of safety provision. Comparing to the first-best, uniform taxation decreases the marginal benefit of safety for any given level of safety due to the unequal distribution of after-tax wealth under risk aversion. In other words, saving a life has less value on average ex-ante because imperfect taxation lowers the average utility in the society. Uniform taxation also increases the marginal cost of safety provision because the average marginal utility of consumption is higher (under prudence) due to suboptimal financing. As a result, saving a life is more costly on average ex-ante, consistent with the Pigou conjecture. Combining the two effects, less safety is provided under uniform taxation than in the first-best.

To illustrate the result with a specific and extreme example, consider two individuals with wealth 1000 and 10 respectively. They both have the same CRRA utility $u(c) = \frac{c^{0.5}}{0.5}$ and survival function $p(G) = \frac{G}{1+G}$. In the first best, the rich is taxed 516.7, and the poor is given a subsidy of 473.3. Therefore, the total investment in public safety is 43.5. Under uniform taxation, each is taxed 7.1, and the total investment on public safety is now 14.2, which is only one-third of the level of safety in the first-best.

3.2 First Best and Income Tax Comparison

Remark 1. *Under wealth heterogeneity, the optimal level of public safety in the first-best could be above, below or equal to the level under income tax.*

Table 1 presents simulations of the optimal public safety provision under three specific cases with CRRA and CARA utility. With CRRA utility, the optimal level is the same under first-best and income tax. With CARA utility, the level of provision may be higher or lower in first-best than with income tax given the degree of risk aversion (parameter α in the utility function).

Table 1 Simulations of the optimal public safety under wealth heterogeneity

Utility		CRRA	CARA	
Parameter value		$\gamma = 0.5$	$\alpha = 0.02$	$\alpha = 0.001$
Tax Rate	t_1	276.64	580.727	272.87
	t_2	-223.354	80.727	-227.13
	$\tau(w_1 + w_2)$	53.286	493.5	51
		$G_F^* = G_I^*$	$G_F^* > G_I^*$	$G_F^* < G_I^*$

Note: Simulated in Mathematica. $p(x) = \frac{x}{1+x}$, CRRA utility $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, CARA utility $u(x) = \frac{1-e^{-\alpha x}}{\alpha}$, $w_1 = 1000$ and $w_2 = 500$.

In the following, we further study the case of CRRA utility.

Remark 2. *Under wealth heterogeneity, if the utility function satisfies CRRA, then the optimal level of public safety in the first-best is always the same as that with income taxation ($G_F^* = G_I^*$).*

Proof. Under CRRA, $u'(c) = c^{-\gamma}$ and the fear of ruin coefficient $FR(c) = \frac{u(c)}{u'(c)} = \frac{c}{1-\gamma}$ is linear in c.

Substituting the utility function into equation 3.4 and 3.6, we get

First-Best:

$$\frac{p(G_F^*)}{p'(G_F^*)} = \frac{w_1 + w_2 - (t_1^* + t_2^*)}{1-\gamma} \tag{3.9}$$

Income Tax:

$$\frac{p(G_I^*)}{p'(G_I^*)} = \frac{w_1 + w_2 - \tau^*(w_1 + w_2)}{1-\gamma} \tag{3.10}$$

Notice that both equations hold when $\tau^*(w_1 + w_2) = t_1^* + t_2^*$. □

The linear fear of ruin property of CRRA utility is instrumental to this result. Notice that with CRRA utility, the RHS of equation 3.10 indicates that only the sum of wealth matters for determining the optimal level of public safety. Therefore, if the sum of wealth remains the same, optimal level of public safety would always coincide in the first-best and in income tax, regardless of how the wealth is distributed.

4 Mortality Risk Heterogeneity

In this section, we consider individual heterogeneity on survival probability $p_i(G)$ (or risk heterogeneity $1 - p_i(G)$) assuming $p_1(G) > p_2(G)$. The social planner's problems can be written as follows:

First-Best:

$$\max_{G_F, t_1, t_2} p_1(G_F)u(w - t_1) + p_2(G_F)u(w - t_2) \quad \text{s.t.} \quad G_F = t_1 + t_2 \tag{4.1}$$

Uniform Tax:

$$\begin{aligned} \max_{G_U, t_U} & (p_1(G_U) + p_2(G_U))u(w - t_U) \\ \text{s.t.} & \quad G_U = 2t_U \end{aligned} \tag{4.2}$$

Income Tax:

$$\begin{aligned} \max_{G_I, \tau} & (p_1(G_I) + p_2(G_I))u(w(1 - \tau)) \\ \text{s.t.} & \quad G_I = 2\tau w \end{aligned} \tag{4.3}$$

Note that income tax is equivalent to uniform tax in this scenario as there is no heterogeneity in wealth. Indeed, one can always set $\tau = \frac{t_U}{w}$ to have $w(1 - \tau) = w - t_U$ and obtain $G_I = G_U$. Therefore, we just focus our analysis on the uniform tax case.

Rearranging the focs we get:

First-Best:

$$\begin{aligned} \frac{p_1(G_F^*) + p_2(G_F^*)}{p_1'(G_F^*) + p_2'(G_F^*)} = & \underbrace{\frac{u(w - t_1^*)}{u'(w - t_1^*)} + \frac{u(w - t_2^*)}{u'(w - t_2^*)}}_{U_{FB}^*} \\ & + \underbrace{\frac{p_2(G_F^*)}{p_1'(G_F^*) + p_2'(G_F^*)}}_{A} \underbrace{\frac{u(w - t_1^*) - u(w - t_2^*)}{u'(w - t_1^*)}}_{B} \underbrace{\left(\frac{p_1'(G_F^*)}{p_1(G_F^*)} - \frac{p_2'(G_F^*)}{p_2(G_F^*)} \right)}_{C} \end{aligned} \tag{4.4}$$

$$p_1(G_F^*)u'(w - t_1^*) = p_2(G_F^*)u'(w - t_2^*) \tag{4.5}$$

Uniform Tax:

$$\frac{p_1(G_U^*) + p_2(G_U^*)}{p_1'(G_U^*) + p_2'(G_U^*)} = \underbrace{\frac{2u(w - t_U^*)}{u'(w - t_U^*)}}_{U_{Uni}^*} \tag{4.6}$$

Again, we are interested in comparing G_F^* and G_U^*. As the LHS of equations 4.4 and 4.6 are of the same form and are increasing functions of G, we only need to examine the RHS of the equations.

Denote $\hat{t_U}$ and $\hat{U}_{Uni}$ such that $\hat{t_U} = \frac{t_1^* + t_2^*}{2}$ and $\hat{U}_{Uni} = \frac{2u(w - \hat{t_U})}{u'(w - \hat{t_U})}$. If $\frac{u}{u'}$ is weakly convex, $U_{FB}^* \geq \hat{U}_{Uni}$. Given our assumptions on the functional forms, we know that $A > 0$ and $B > 0$. Hence if we can pin down the sign of C, we can rank G_F^* and G_U^*. If $\frac{p_1'(G_F^*)}{p_1(G_F^*)} \geq \frac{p_2'(G_F^*)}{p_2(G_F^*)}$, it follows that $C \geq 0$. Thus, the RHS of 4.4 is greater than the RHS of 4.6 when $t_U^* = \hat{t_U}$. Therefore, it must be that $t_U^* < \hat{t_U}$ and $G_U^* < G_F^*$. If $\frac{p_1'(G_F^*)}{p_1(G_F^*)} < \frac{p_2'(G_F^*)}{p_2(G_F^*)}$, then $C < 0$. Under $\frac{u}{u'}$ weakly concave, as $U_{FB}^* \leq \hat{U}_{Uni}$, the RHS of 4.4 must be lower than 4.6, thus $t_U^* > \hat{t_U}$ and $G_U^* > G_F^*$.

The risk heterogeneity may come from two sources: baseline risk and risk reduction. More specifically, baseline risk refers to the individual risk prior to the implementation of the public safety project, and risk reduction refers to the individual benefit from the project. In the remainder of this section, we separately analyze heterogeneous baseline risk and heterogeneous risk reduction.

Heterogeneous Baseline Risk

With heterogeneous baseline risk, agents have different baseline survival probability p_i, but receive the same level of benefit from the public safety project $\varepsilon(G)$. The survival function could be expressed as $p_i(G) = p_i + \varepsilon(G)$, with $\varepsilon(\cdot) < 1 - \min\{p_1, p_2\}$, $\varepsilon(\cdot) > 0$, $\varepsilon'(\cdot) > 0$, and $\varepsilon''(\cdot) \leq 0$.

Proposition 2. *Under heterogeneous baseline risk ($p_i(G) = p_i + \varepsilon(G)$), if fear of ruin is weakly concave, the optimal level of public safety is lower in the first-best than with uniform or income tax ($G_F^* < G_U^* = G_I^*$).*

Proof. Assuming $p_1 > p_2$, then

$$\frac{p_1'(G_F^*)}{p_1(G_F^*)} = \frac{\varepsilon'(G_F^*)}{p_1 + \varepsilon(G_F^*)} < \frac{\varepsilon'(G_F^*)}{p_2 + \varepsilon(G_F^*)} = \frac{p_2'(G_F^*)}{p_2(G_F^*)}$$

Thus $C < 0$. We know from the above analysis that with $\frac{u}{u'}$ weakly concave (e.g. CRRA utility), $G_F^* < G_U^* = G_I^*$ when $C < 0$. □

We show in Table 2 that if utility is CARA, the optimal level of public safety can be greater or lower in the first-best than under uniform or income tax.

Table 2 Simulation for heterogeneous baseline risk with CARA utility

Utility		CARA	
Wealth level		$w = 1000$	$w = 20$
Tax Rate	t_1	667.82	-11.73
	t_2	679.21	13.02
	t	673.23	0.79
		$G_F^* > G_U^* = G_I^*$	$G_F^* < G_U^* = G_I^*$

Note: Simulated in Mathematica. $p(G) = p_i + \frac{0.01G}{1+0.02G}$, $p_1 = 0.5$, $p_2 = 0.3$. CARA utility $U(x) = \frac{1-e^{-\alpha x}}{\alpha}$, $\alpha = 0.02$.

The result displayed in proposition 2 goes in the opposite direction of Pigou's intuition. First-best equalizes the expected marginal utility of individuals by imposing a lower tax on the less-exposed individual (i.e. one with higher baseline survival probability) and a higher tax on the more-exposed individual (i.e. one with lower baseline survival probability). This is in line with the dead-anyway effect (Pratt and Zeckhauser, 1996). Under uniform taxation and weakly concave fear of ruin, uniform taxation may exacerbate this effect, which results in a higher public safety level.

Heterogeneous Risk Reduction

With heterogeneous risk reduction, agents have the same baseline survival probability p, but have different degrees of benefit δ_i from the safety project. The survival function is assumed to be linear in the public safety level, thus $p_i(G) = p + \delta_i G$, $\delta_i < \frac{1-p}{G}$ for any G.

Proposition 3. *Under heterogeneous linear risk reduction ($p_i(G) = p + \delta_i G$), if fear of ruin is weakly convex, the optimal level of safety provision in the first-best is higher than that with uniform or income tax ($G_F^* > G_U^* = G_I^*$).*

Proof. By assumption, $\delta_1 > \delta_2$. Therefore,

$$\frac{p_1'(G_F)}{p_1(G_F)} = \frac{\delta_1}{p + \delta_1 G_F} > \frac{\delta_2}{p + \delta_2 G_F} = \frac{p_2'(G_F)}{p_2(G_F)}$$

Thus $C > 0$ and $G_F^* > G_U^*$ for all utility functions that have $\frac{u}{u'}$ weakly convex. □

Similar to the heterogeneous baseline case, first-best imposes a lower tax on the more-responsive individual (i.e. one with higher risk reduction rate) and a higher tax on the less-responsive. Whereas under uniform taxation and weakly convex fear of ruin, it is optimal to reduce public safety.

5 Distortionary Taxation

In this section, we focus on the distortionary aspect of imperfect taxation. We assume that individuals are identical (so that we drop the individual i's index) and that they maximize their utility by choosing the consumption c and labor supply l, subject to the tax rate t. Individuals are assumed to be small, so that they do not take into account the feedback effect of taxation.[10] Following the standard public economics literature, with identical individuals, the first-best is equivalent to uniform lump-sum taxation. Here, we take income tax as the imperfect taxation case.

We solve for the optimal level of public safety by backward induction. In the second stage, individuals take the tax rate as given and maximize their utilities subject to their budget constraints. The individual's problems in the first-best and under income tax are:

First-best:

$$\begin{aligned} &\max_{c_t, l_t} \quad p(G_F)u(c_t, l_t) \\ &\text{s.t.} \quad c_t = wl_t - t \end{aligned} \tag{5.1}$$

Income tax:

$$\begin{aligned} &\max_{c_\tau, l_\tau} \quad p(G_I)u(c_\tau, l_\tau) \\ &\text{s.t.} \quad c_\tau = wl_\tau(1-\tau) \end{aligned} \tag{5.2}$$

The optimal individual decision in the first-best is characterized by $c_t^*(t)$, $l_t^*(t)$: $-\frac{u_l^*}{u_c^*} = w$, where $u_c^* = \frac{\partial u(c_t^*, l_t^*)}{\partial c_t}$ and $u_l^* = \frac{\partial u(c_t^*, l_t^*)}{\partial l_t}$. Under income tax, the optimal individual decision is $c_\tau^*(\tau), l_\tau^*(\tau)$ such that $-\frac{u_l^*}{u_c^*} = w(1-\tau)$.

In the first stage, the social planner determines the optimal level of taxation subject to the revenue requirement for public safety provision. The social planner's problems are respectively:

First-best:

$$\begin{aligned} &\max_{G_F, t} \quad Hp(G_F)u\Big(c_t^*(t), l_t^*(t)\Big) \\ &\text{s.t.} \quad G_F = Ht \end{aligned} \tag{5.3}$$

[10] Individuals take the level of public safety as given. That is, they do not take into consideration that their labor supply level could influence the total amount of safety.

Income tax:

$$\begin{aligned} \max_{G_I,\tau} \quad & Hp(G_I)u\Big(c^*_\tau(\tau), l^*_\tau(\tau)\Big) \\ \text{s.t.} \quad & G_I = Hwl^*_\tau(\tau)\tau \end{aligned} \tag{5.4}$$

Rearranging the focs from the planner's problems, we get:

First-best:

$$\frac{p(G^*_F)u_c\Big(c^*_t(t), l^*_t(t)\Big)}{Hp'(G^*_F)u\Big(c^*_t(t), l^*_t(t)\Big)} = 1 \tag{5.5}$$

Income tax:

$$\frac{p(G^*_I)u_c\Big(c^*_\tau(\tau), l^*_\tau(\tau)\Big)}{Hp'(G^*_I)u\Big(c^*_\tau(\tau), l^*_\tau(\tau)\Big)} = 1 \times (1 + \varepsilon_{l\tau^*}) \tag{5.6}$$

where $\varepsilon_{l\tau^*} = \frac{\partial l}{\partial \tau} / \frac{l}{\tau}$ denotes the labor supply elasticity of income tax.

Observe that equation 5.5 is the Samuelson condition of public good provision, where the sum of the marginal rate of substitution (MRS) between the public good and private consumption equals to the marginal rate of transformation (MRT). Equation 5.6 is thus the modified Samuelson's rule for distortionary taxation, where the sum of MRS is equal to MRT multiplied by a term denoted as the marginal cost of public funds (MCPF). Here, $MCPF = 1 + \varepsilon_{l\tau^*}$ and its value determines whether the first-best public safety provision G^*_F is greater or lower than that with a income tax G^*_I: for $\varepsilon_{l\tau} > 0$, $MCPF > 1$, $G^*_I < G^*_F$ and vice versa.

The value of $\varepsilon_{l\tau}$ depends on the properties of the utility function. Here we consider two cases: tangible labor effort (i.e. commensurable with wealth) $u(c, l) = u(c - e(l))$, and non-tangible labor effort $u(c, l) = u(c) - e(l)$ (with $e(l) > 0$, $e'(l) > 0$, $e''(l) > 0$).

Proposition 4. *Under distortionary tax with identical individuals,*

1. *if labor effort is tangible $u(c, l) = u(c - e(l))$, the elasticity of labor supply is negative ($\varepsilon_{l\tau^*} < 0$) and the optimal level of public safety is lower under income tax tax than in the first-best ($G^*_F = G^*_U > G^*_I$).*

2. *if labor effort is non-tangible $u(c, l) = u(c) - e(l)$, the elasticity of labor supply is positive (negative) ($\varepsilon_{l\tau^*} > 0 (< 0)$) if the level of relative risk aversion is greater (lower) than 1 ($R > 1 (< 1)$), and the optimal level of public safety is lower (higher) under income tax than in the first-best ($G^*_F = G^*_U > (<) G^*_I$).*

Proof. As $\varepsilon_{l\tau^*} = \frac{\partial l}{\partial \tau} / \frac{l}{\tau}$ and $\frac{l^*_t(\tau)}{\tau^*} > 0$, to obtain the sign of $\varepsilon_{l\tau^*}$, we just need to sign $\frac{\partial l^*(\tau)}{\partial \tau}$.

1. Tangible labor effort $u(c, l) = u(c - e(l))$, $c = wl(1 - \tau)$

$$\frac{\partial l^*(\tau)}{\partial \tau^*} = -\frac{\frac{\partial^2 u}{\partial l \partial \tau}}{\frac{\partial^2 u}{\partial^2 l}} = -\frac{w}{e''(l^*(\tau))} \tag{5.7}$$

By assumption, $e''(l) > 0$, then $\frac{\partial l^*(\tau)}{\partial \tau^*} < 0 \implies \varepsilon_{l\tau^*} < 0 \implies G_F^* > G_I^*$.

2. Non-tangible labor effort $u(c,l) = u(c) - e(l)$, $c = wl(1-\tau)$

$$\frac{\partial l^*(\tau)}{\partial \tau^*} = \frac{u''(c^*(\tau))w^2 l^*(1-\tau) + u'(c^*(\tau))w}{u''(c^*(\tau))w^2(1-\tau)^2 - e''(l^*(\tau))} \tag{5.8}$$

By assumption, the denominator is negative. In this case, if the relative risk aversion coefficient $R(c^*(\tau)) = -c^*(\tau)\frac{u''(c^*(\tau))}{u'(c^*(\tau))} < 1$, then the numerator of the RHS of equation 5.8 is positive, which implies $\varepsilon_{l\tau^*} < 0$, and it follows that $G_F^* > G_I^*$. If $R(c^*(\tau)) \geq 1$, then $\varepsilon_{l\tau^*} \geq 0$ and $G_F^* \leq G_I^*$.

□

Under CRRA, $R(c^*(\tau)) = \gamma$. As utility is positive, $\gamma < 1$, thus $R(c^*(\tau)) < 1$. Therefore, under CRRA, even with a non-tangible cost of labor, we have $G_F^* = G_U^* > G_I^*$. However, with other utility functions, it may occur that $G_F^* \leq G_I^*$. For example, for CARA, $u(w) = \frac{1-e^{-\alpha w}}{\alpha}$, we have $R = \alpha w$, which is greater than one if $\alpha > \frac{1}{w}$.

How can we explain the ambiguous result under the non-tangible labor effort? Taxation creates both substitution and income effects, with opposite effects on the optimal labor supply. The substitution effect reduces labor supply (increases leisure) because, for a given total income, it is less profitable to work. However, a higher tax rate also decreases total income and increases the marginal utility of income. Therefore, the income effect increases labor supply because the return of an extra unit of labor has a higher marginal value. With $R > 1$, the curvature of the utility function is large. Thus, a small decrease in wealth would imply a large increase in the marginal utility. As a result, the income effect is larger than the substitution effect. When labor effort is "tangible", the curvature of the utility function does not play any role in optimal decision making ($e'(l^*) = w(1-\tau^*)$). Whereas when labor effort is separable from the utility of wealth, the curvature of utility function effectively affects labor supply ($e'(l^*) = w(1-\tau^*)u'(w(1-\tau^*)l^*)$. Thus, only in the "non-tangible" case, the ambiguous result can occur.[11]

6 Inequalities and Public Safety

As is documented in the World Inequality Report 2018, wealth and income inequalities within world regions varies greatly and have been increasing in nearly all countries since 1980 (Alvaredo et al., 2018). Moreover, several studies in the public health sector document significant level of inequality in mortality risks in almost all countries caused both by differences in socioeconomic status and health behaviors (Mackenbach et al., 2008; Laaksonen et al., 2007). In this section, we ask: How does wealth and risk inequalities affect public safety? And to which extent the relationship between inequality and public safety varies with tax system imperfections?

[11] In the tangible labor effort case, there is no income effect of taxation. Thus, the substitution effect always reduces labor supply with an increase in the tax rate.

6.1 Wealth Inequality

Here we consider the model in section 3, and we assume $w_1 = (1+\eta)\bar{w}$, $w_2 = (1-\eta)\bar{w}$, where $\bar{w}$ denotes the average wealth. Here $\eta \in [0, 1)$ measures wealth inequality with $\eta = 0$ indicating perfect equality.

Proposition 5. *An increase in wealth inequality does not affect the first-best optimal level of public safety, but reduces the optimal safety level under uniform taxation if* $u''' \geq 0$.

Proof. In the first-best, the optimal level of taxation is characterized by $(1+\eta)\bar{w} - t_1^* = (1-\eta)\bar{w} - t_2^*$ from equation 3.4. A change in wealth inequality can be expressed as $\eta' = \eta + \Delta\eta$. The optimality condition gives $(1+\eta+\Delta\eta)\bar{w} - T_1^* = (1-\eta-\Delta\eta)\bar{w} - T_2^*$. Thus, it is straightforward that $t_1^* = T_1^* - \Delta\eta\bar{w}$ and $t_2^* = T_2^* + \Delta\eta\bar{w}$. It follows that $T_1^* + T_2^* = t_1^* + t_2^*$.

For uniform taxation, we can rewrite equation 3.5 as a function of η:

$$F(t_U^*, \eta) \equiv p(2t_U^*)(u_1' + u_2') - 2p'(2t_U^*)(u_1 + u_2) = 0 \tag{6.1}$$

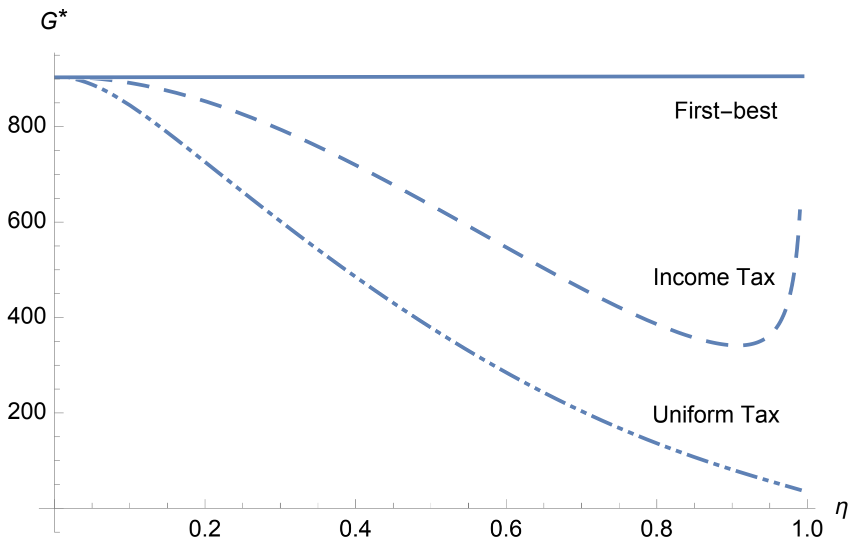
where $u_1 = u((1+\eta)\bar{w} - t_U^*)$ and $u_2 = u((1-\eta)\bar{w} - t_U^*)$. Applying the Implicit Function Theorem, it is easy to obtain that

$$\frac{\mathrm{d}t_U^*}{\mathrm{d}\eta} = -\frac{F_\eta}{F_{t_U^*}} < 0 \tag{6.2}$$

if $u''' \geq 0$. Therefore, assuming prudence, t_U^* decreases in η.

□

Fig. 2 Effect of wealth inequality on the optimal level of public safety under the three tax systems



Note: Simulated in Mathematica. $p(x) = 0.2 + \frac{0.02x}{1+0.04x}$, CARA utility $u(x) = \frac{1-e^{-\alpha x}}{\alpha}$, $\alpha = 0.01$ and $w = 1000$.

Figure 2 illustrates proposition 5: in the first-best, increasing inequality does not affect the optimal level of safety; under uniform tax, the level is monotonically decreasing with increasing inequality. The first part of proposition 5 is analogous to the well known result of private provision of public good: wealth redistribution among contributors does not change the equilibrium supply of public good (Bergstrom et al., 1986).

Figure 2 also shows that with income taxation, increasing wealth inequality may not monotonically change the optimal level of public safety. For example, with a specific CARA utility function, increasing wealth inequality first decreases and then increases the optimal level of public safety. If utility satisfies CRRA, given the result from remark 2, the optimal level of public safety remains unchanged regardless of the degree of inequality.

6.2 Risk Inequality

Here we consider the model in section 4, and we separately analyze the effect of baseline risk inequality and risk reduction inequality. For baseline risk inequality, we set $p_1^b(G) = (1+\eta)\bar{p} + \varepsilon(G)$ and $p_2^b(G) = (1-\eta)\bar{p} + \varepsilon(G)$. For risk reduction inequality, we set $p_1^r(G) = p + (1+\eta)\bar{\delta}G$ and $p_2^r(G) = p + (1-\eta)\bar{\delta}G$. As before, η denotes the degree of inequality and $\eta \in [0,1)$.

Proposition 6. *An increase in risk inequality (both baseline risk and risk reduction) does not affect the optimal level of public safety under uniform and income tax.*

Proof. For baseline risk inequality, equation 4.6 can be rewritten as:

$$\frac{\bar{p} + \varepsilon(G_U^*)}{\varepsilon'(G_U^*)} = \frac{2u(w - t_U^*)}{u'(w - t_U^*)}.$$

As the foc of uniform tax is independent of η, G_U (and equivalently G_I) remains constant regardless of η.

For risk reduction inequality, the LHS of 4.6 can be written as $\frac{p + \bar{\delta}G_U^*}{\bar{\delta}G_U^*}$, which is also independent of η. □

The simulations show that in the first-best, however, increasing risk inequality affects the optimal public safety level and magnifies the gap between the level in the first-best and under uniform taxation. Figure 3 shows the optimal safety level with respect to the baseline risk inequality and risk reduction inequality.

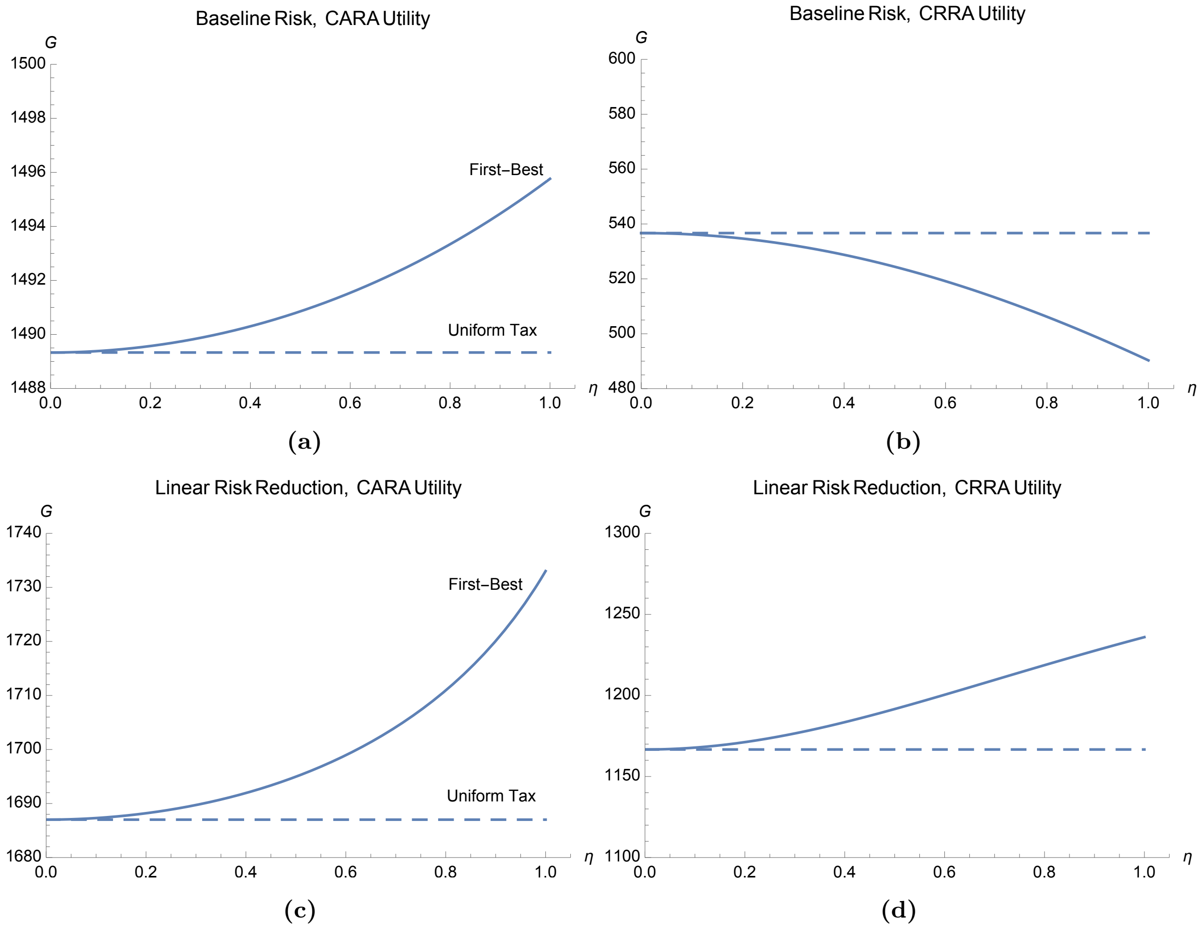
7 Policy Implications

7.1 VSL and Distributional Weights under Imperfect Taxation

In practice, several policy-making agencies that implement public safety projects, e.g. the U.S. EPA and the U.S. DOT, commonly use the VSL to monetize mortality risk reduction benefit. The recommended VSL is usually obtained from meta-analysis of VSL estimates from stated preference, revealed preference and hedonic wage studies (U.S. Environmental Protection Agency, 2016b). This VSL usually represents a population average of individual VSLs and will be referred to as "average VSL" further on.

Recall from section 2 that the necessary condition to achieve optimality in public safety provision is $\sum_{i=1}^{H} p_i'(G^*) VSL_i = 1$, where $VSL_i = \frac{u(w_i - t_i^*)}{p_i(G^*)u'(w_i - t_i^*)}$. Observe that when $p_i'(G^*)$ is independent from VSL_i, the efficiency condition can be rewritten as $\frac{1}{n}\sum_{i=1}^{H} VSL_i = \frac{1}{\sum_{i=1}^{H} p_i'(G^*)}$, which equates the

Fig. 3 Effect of risk inequality on the optimal safety level



Note: Simulated in Mathematica. $p_i(x) = p_i + \frac{0.02x}{1+0.04x}$, $p_1 = (1+\eta)\bar{p}$, $p_2 = (1-\eta)\bar{p}$, $\bar{p} = 0.25$, CARA utility $u(x) = \frac{1-e^{-\alpha x}}{\alpha}$, $\alpha = 0.02$, CRRA utility $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, $\gamma = 0.5$ and $w = 1000$.

average VSL to the social marginal cost of saving a life. Therefore, average VSL can determine the optimal level of public safety if taxation is perfect and $p_i'(G^*)$ is independent of VSL_i. In the absence of either condition, however, the average VSL can lead to an over- or under-valuation of the social value of public safety. One way to address this concern in practice is to incorporate "distributional weights" into BCA (Adler, 2016).

Currently, the official guidelines for BCA in the UK recommend using distributional weights that can be expressed as the marginal utility of each quintile as a percentage of average marginal utility (Treasury, 2003). However, this weighting scheme only accounts for income inequalities and does not consider other inequalities, such as risk inequality. Moreover, it does not explicitly address the question of imperfect taxation. In the following, we present a simple exercise of re-expressing the optimality conditions in terms of a weighted VSL and discuss the weighting rules.

We rewrite the optimality conditions as follows

$$\sum_{i=1}^{H} \lambda_i p_i'(G) VSL_i = 1, \tag{7.1}$$

where λ_i is the corresponding weight assigned to each individual. The weights vary with the tax system and sources of heterogeneity. Table 3 shows the weights in each case.

Table 3 VSL weights under different taxation systems and heterogeneities

	Wealth Heterogeneity	**Risk Heterogeneity**
First-Best	1	1
Uniform Tax	$\frac{u'(w_i - t_U^*)}{\frac{1}{H}\sum_j u'(w_j - t_U^*)}$	$\frac{p_i(G_U^*)}{\frac{1}{H}\sum_j p_j(G_U^*)}$
Income Tax	$\frac{u'(w_i(1-\tau^*))}{\sum_k \frac{w_k}{\sum_j w_j} u'(w_k(1-\tau^*))}$	$\frac{p_i(G_I^*)}{\frac{1}{H}\sum_j p_j(G_I^*)}$

It is straightforward that there is not one set of weights that can be applied to all cases. In the first-best, no weight is needed, of course. In the case of wealth heterogeneity, under uniform tax, the weights are similar to the recommended distributional weights in the UK ($\lambda_i = \frac{u'(w_i - t_U^*)}{\frac{1}{H}\sum_j u'(w_j - t_U^*)}$). Under income tax, the weights can be expressed as the marginal utility as a percentage of the sum of wealth weighted ($\frac{w_k}{\sum_j w_j}$) marginal utilities. Moreover, under risk heterogeneity with uniform and income taxation, the weights should be the individual survival probability as a percentage of the population average survival probability ($\lambda_i = \frac{p_i(G_U^*)}{\frac{1}{H}\sum_j p_j(G_U^*)}$).

7.2 VSL Transfer

VSL is used in BCA for a variety of policy evaluations. However, it is costly to conduct case-specific VSL studies. Thus, a common practice is to take the VSL value in some case studies and quantitatively adjust the value to fit the policy context, known as "benefit transfers" (U.S. Environmental Protection Agency, 2011). A common practice is to adjust VSL by the income elasticity of the populations under study (Hammitt and Robinson, 2011). Our analysis suggests that, in addition to income elasticity, inequality and tax system imperfections also need to be considered.

Meta-analysis of wage-risk studies has shown that the VSL estimates of developed countries (e.g. U.S., UK) can be more than ten times the estimates of middle-income countries (e.g. China) (Viscusi and Aldy, 2003). Moreover, the extrapolated VSL under income adjustment for low-income countries (e.g. Kenya, Ethiopia) could be 50 times lower than that of the U.S. (Hammitt and Robinson, 2011). Although these values already raise controversy, we argue there are two reasons to even further adjust these estimates: the inequality of wealth and risk as well as the imperfection in the taxation systems.

Proposition 5 and 6 shows that a higher degree of wealth or mortality risk inequality within the population could further increase or decrease the level of optimal public safety (i.e. the gap between first-best and imperfect taxation optimal level of safety widens with increasing inequality). Furthermore, in the case of wealth inequality, the taxation system in place would also affect the degree of adjustment in the optimal safety level.

7.3 The Marginal Cost of Public Funds

The marginal cost of public funds (MCPF) measures the loss incurred by raising additional revenues to finance government spending. However, no consensus has yet been reached on the value of MCPF (Dahlby, 2008). In practice, agencies adopt different values of MCPF in their guidelines for BCA, but they are usually greater than unity. For example, the U.S. Office of Management and Budget (OMB) recommends using an MCPF of 1.25 (Office of Management and Budget, 1992, article 11), the European Union uses a default MCPF of 1 in the absence of national guidelines (Florio et al., 2008) and the French government recommends using a median value of 1.2 (Quinet, 2013). Our analysis indicates that there is no solid scientific rationale for these numbers in the context of public safety projects.[12]

Proposition 4 shows that the MCPF can be greater or lower than unity depending on the labor supply elasticity. Although it is a theoretical possibility in accordance with Atkinson and Stern (1974) that the income effect may dominate the substitution effect resulting in a positive labor supply elasticity, there is little empirical evidence of such occurrence (Meghir and Phillips, 2010). However, Manski (2014) argues that the consensus in the empirical literature may be an artifact of the strong assumptions made in the models.[13] He states that without the knowledge of income-leisure preference, one cannot predict how labor effort may change with the tax rate.

[12]There are two competing approaches to the MCPF, namely the Dasgupta-Stiglitz-Atkinson-Stern (DSAS) tradition, and the Pigou-Harberger-Browning (PHB) tradition (Dahlby, 2008). Our analysis follows the DSAS approach, where the social planner's budget is balanced.

[13]Manski (2014, p.147) wrote, "Examining the models of labor supply used in empirical research, I have become concerned that the prevailing consensus on the sign of uncompensated elasticities may be an artifact of model specification rather than an expression of reality". He pointed out that the two assumptions generally made in the usual empirical models, non-backward-bending labor supply functions and homogeneous response of labor supply to net wage across populations, may lead mechanically to the positive labor supply elasticity of tax.

A Appendix

A.1 Second Order Conditions

For the general framework, we assume that the second order conditions (socs) are satisfied.

For the wealth heterogeneity and distortionary taxation case, the SOCs of the social planner's problems (3.1, 3.2, 3.3, 5.3, and 5.4) are always satisfied under the assumptions made.

For the risk heterogeneity case, the SOCs of the uniform and income tax problem (4.2, 4.3) are always satisfied. For the first-best (4.1), in order to have the SOC satisfied, the Hessian of 4.1 need to be negative definite. Denoting $f(t_1^*, t_2^*) \equiv p_1(t_1^* + t_2^*)u(w - t_1^*) + p_2(t_1^* + t_2^*)u(w - t_2^*)$, this would require that $\frac{\partial^2 f(t_1^*, t_2^*)}{\partial t_1^2} < 0$, $\frac{\partial^2 f(t_1^*, t_2^*)}{\partial t_2^2} < 0$ and $\begin{vmatrix} \frac{\partial^2 f(t_1^*, t_2^*)}{\partial t_1^2} & \frac{\partial^2 f(t_1^*, t_2^*)}{\partial t_1 \partial t_2} \\ \frac{\partial^2 f(t_1^*, t_2^*)}{\partial t_1 \partial t_2} & \frac{\partial^2 f(t_1^*, t_2^*)}{\partial t_2^2} \end{vmatrix} > 0$. The first two conditions are easy to show. For the last condition, denote: $A_1 = p_1'' u_1, A_2 = p_2'' u_2, B_1 = p_1' u_1', B_2 = p_2' u_2', C_1 = p_1 u_1'', C_2 = p_2 u_2''$. If $(A_1 + A_2)(C_1 + C_2) - (B_1 - B_2)^2 - 2B_1C_2 - 2B_2C_1 + C_1C_2 > 0$, then the SOC is satisfied globally.

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